

viernes, 24 de junio de 2022

Cálculo Integral

$$L = \int_a^b \sqrt{1 + \left(\frac{d}{dx}(y(x)) \right)^2} dx$$

$$\left. \begin{array}{l} x = x(t) \\ y = y(t) \end{array} \right\} \Rightarrow \begin{array}{l} a = x(t_1) \\ b = x(t_2) \end{array}$$

$$\frac{dy}{dx} = \frac{\left(\frac{dy}{dt} \right)}{\left(\frac{dx}{dt} \right)}, \quad dx = \frac{dx(t)}{dt} \cdot dt$$

$$L = \int_{t_1}^{t_2} \sqrt{1 + \left[\frac{\left(\frac{dy}{dt} \right)}{\left(\frac{dx}{dt} \right)} \right]^2} \cdot x'(t) \cdot dt$$

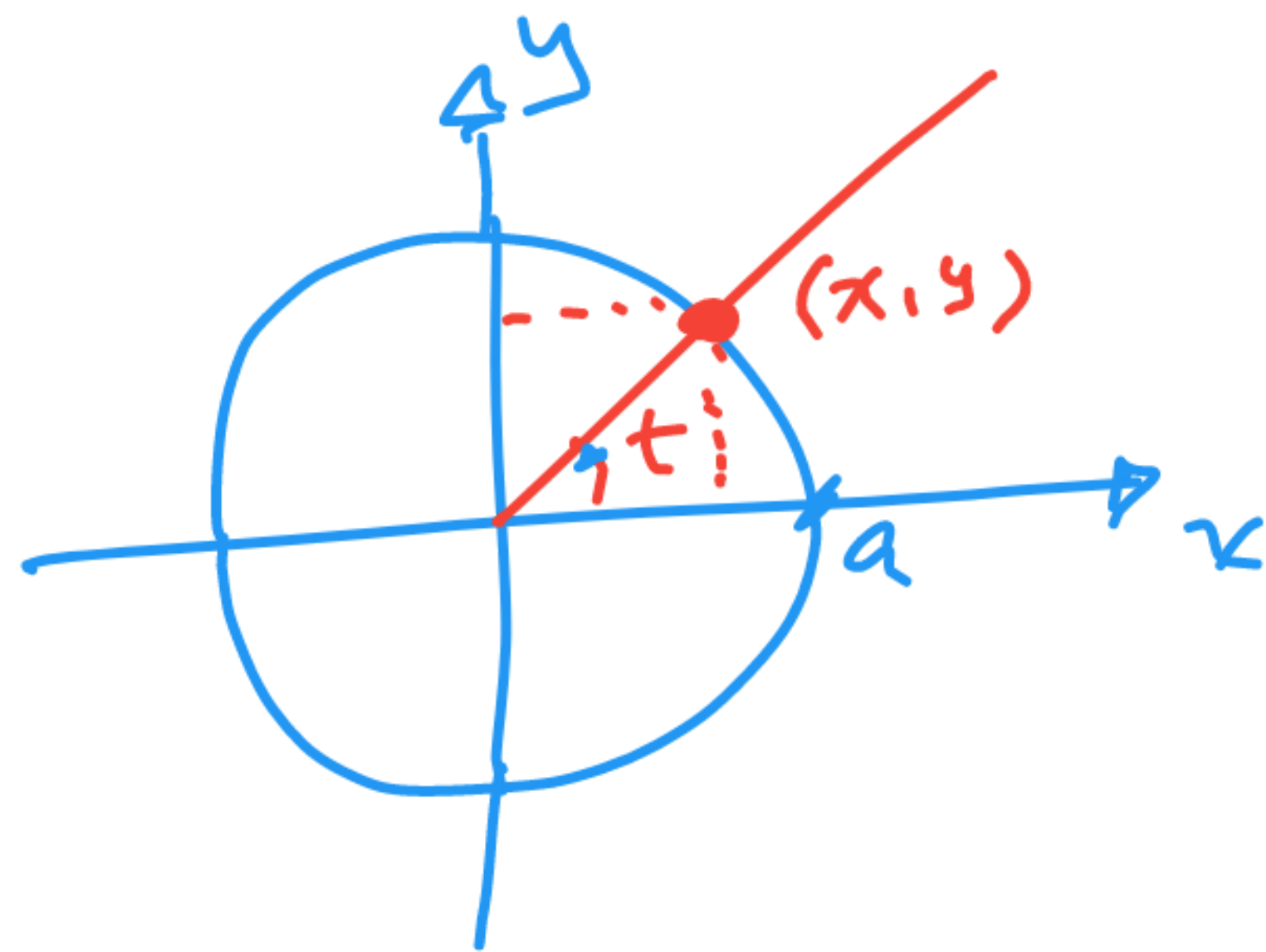
$$L = \int_{t_1}^{t_2} \sqrt{\frac{(x'(t))^2 + (y'(t))^2}{(x'(t))^2}} \cdot x'(t) dt$$

$$L = \int_{t_1}^{t_2} \sqrt{(x'(t))^2 + (y'(t))^2} \cdot \frac{1}{|x'(t)|} \cdot x'(t) dt$$

$$L = \int_{t_1}^{t_2} \sqrt{(x'(t))^2 + (y'(t))^2} \cdot dt, \quad \text{con } x'(t) > 0 \\ \text{en } [t_1, t_2]$$

Exemple :

$$x = a \cos t$$
$$y = a \sin t$$



$$L = 4 \int_0^a \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$$

$$L = 4 \int_{\pi/2}^0 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$$

$$L = 4 \int_0^{\pi/2} \sqrt{a^2 \sin^2 t + a^2 \cos^2 t} dt = 4a \int_0^{\pi/2} dt = 4a \left(\frac{t}{1} \right)_0^{\pi/2} = 2a\pi$$

$$\text{Si } x=0 \Rightarrow \cos t = 0 \Rightarrow t = \pi/2$$

$$\text{Si } x=a \Rightarrow \cos t = 1 \Rightarrow t = 0$$

$$x(t) = a \cos t$$

$$x'(t) = -a \sin t < 0 \quad \forall t \in [0, \pi/2]$$

Ejercicio de la cicloide pag 283 de Haynard Kong.

$$x(t) = a(t - \sin t) \Rightarrow x'(t) = a(1 - \cos t) > 0, \forall t \in]0, 2\pi[; a > 0$$

$$y(t) = a(1 - \cos t) \Rightarrow y'(t) = a(\sin t)$$

$$L = \int_0^{2\pi} \sqrt{a^2(1 - \cos t)^2 + a^2 \sin^2 t} \, dt =$$

Se tiene:

$$\begin{aligned} \sqrt{a^2(1 + \cos^2 t - 2\cos t + \sin^2 t)} &= \sqrt{a^2(2 - 2\cos t)} = \sqrt{2a^2(1 - \cos t)} \\ &= \sqrt{2} a \sqrt{1 - \cos t} = 2a \sqrt{\frac{1 - \cos t}{2}} = \end{aligned}$$

$$= 2a \sqrt{\text{Sen}^2\left(\frac{t}{2}\right)} = 2a \left| \text{Sen}\left(\frac{t}{2}\right) \right|$$

$$t \in [0, 2\pi] \Rightarrow \frac{t}{2} \in [0, \pi] \Rightarrow \left| \text{Sen}\left(\frac{t}{2}\right) \right| = \text{Sen}\left(\frac{t}{2}\right)$$

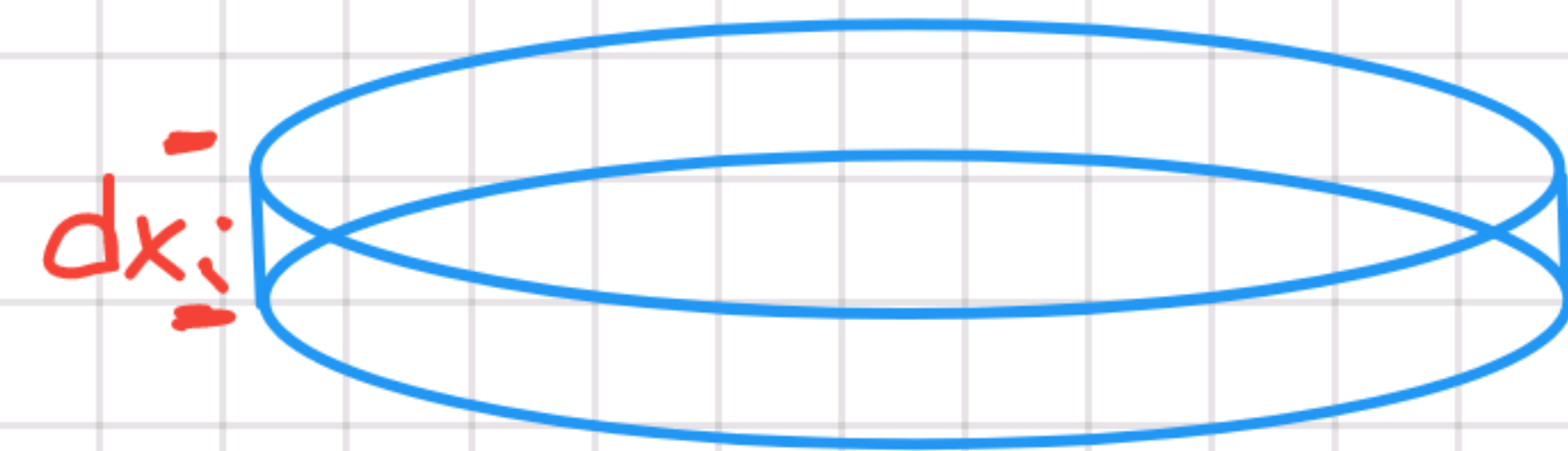
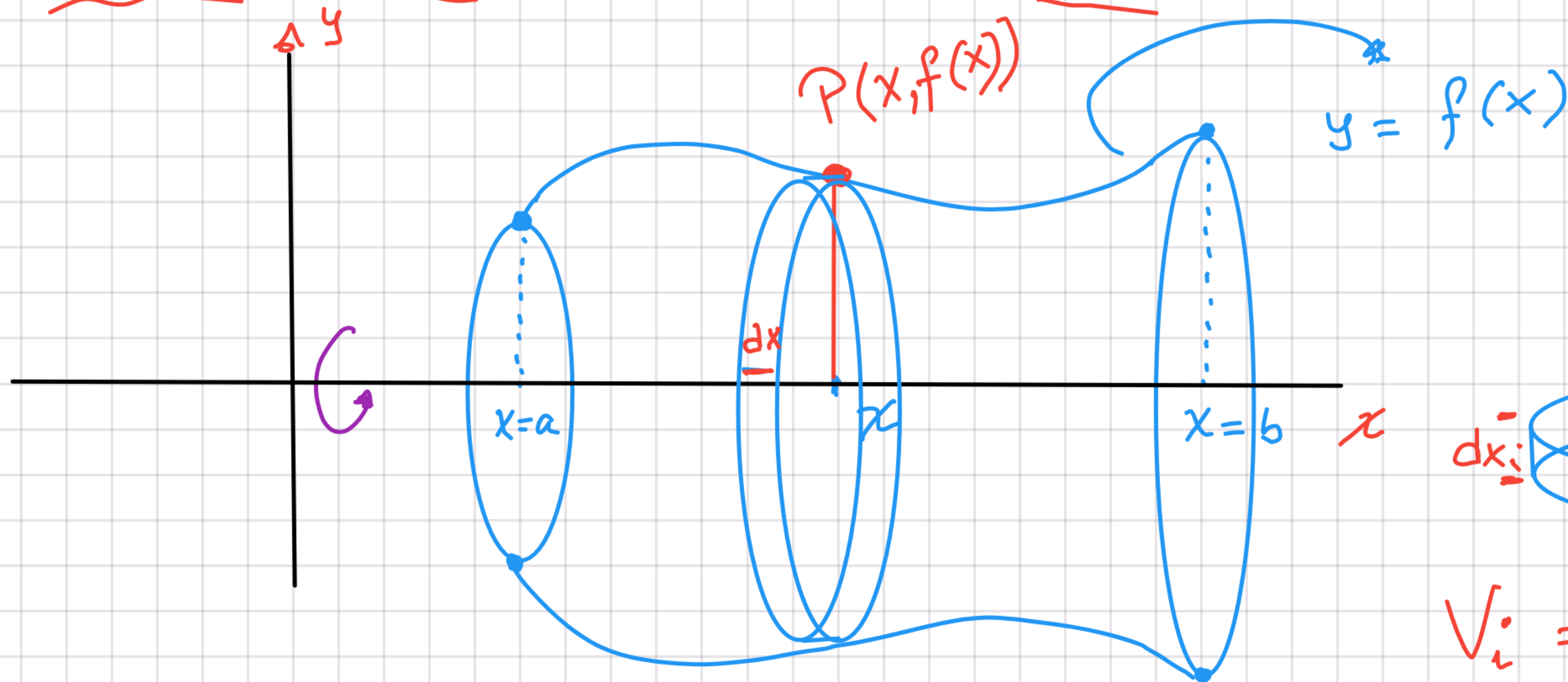
∴

$$\begin{aligned} L &= 2a \int_0^{2\pi} \text{Sen}\left(\frac{t}{2}\right) dt = -4a \cdot \cos\left(\frac{t}{2}\right) \Big|_0^{2\pi} \\ &= -4a [\cos(\pi) - \cos(0)] \\ &= -4a [-1 - 1] = 8a \text{ unidades} \end{aligned}$$

Volumenes de sólidos

Ver pag 293 del libro Cálculo Int. de Maynard Kong.

Volumenes de sólidos de revolución



$$V_i = \pi (f(x_i))^2 \cdot dx_i$$

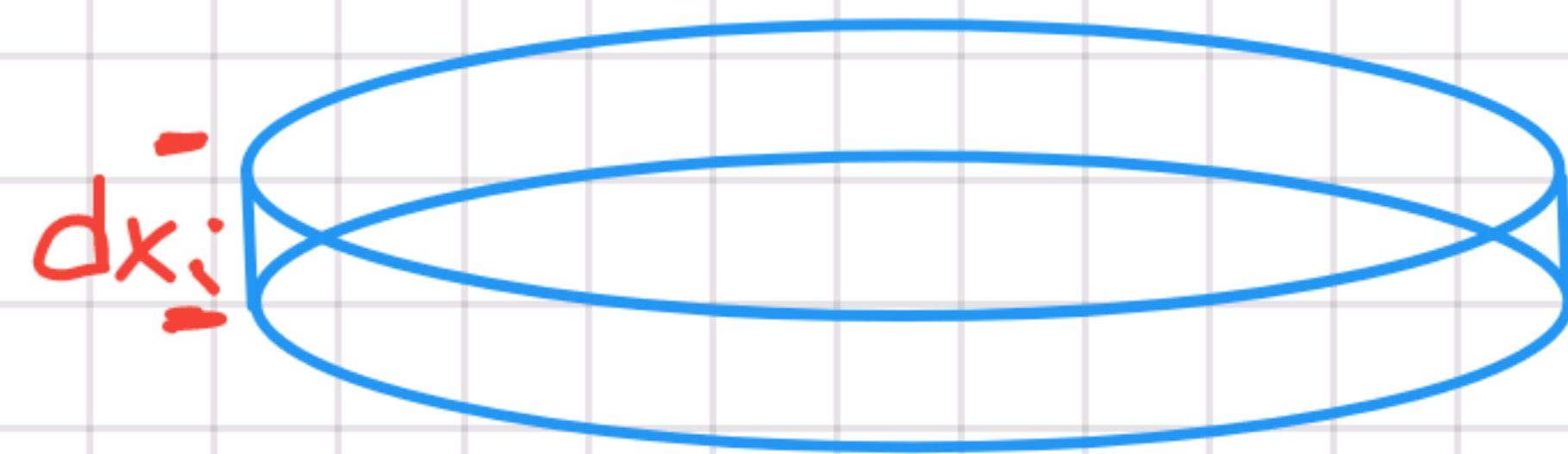
V = volumen del sólido de revolución

$$V = \lim_{n \rightarrow \infty} \sum_{i=1}^n V_i$$

$$V = \lim_{n \rightarrow \infty} \sum_{i=1}^n \pi (f(x_i))^2 \cdot \Delta x_i$$

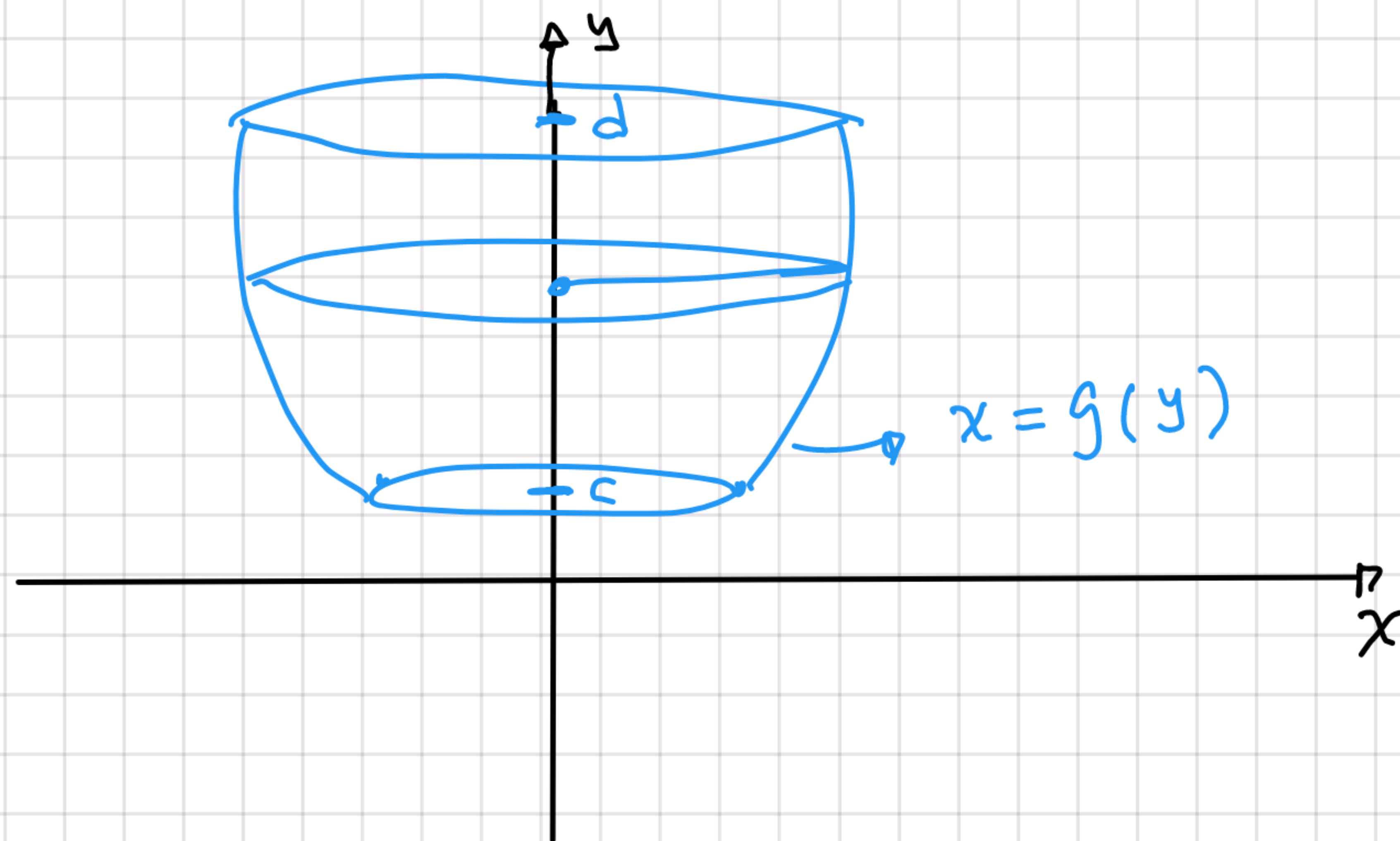
$$V = \pi \left(\lim_{n \rightarrow \infty} \sum_{i=1}^n (f(x_i))^2 \cdot \Delta x_i \right)$$

$$V = \pi \int_a^b [f(x)]^2 dx$$



$$V_i = \pi (f(x_i))^2 \cdot dx_i$$

$$V = \pi \int_a^b [f(x)]^2 dx \text{ u}^3$$



$$V = \pi \int_c^d (g(y))^2 dy \text{ u}^3$$